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Abstract

The present work is the first to formally model inter-outfit strategic cooperation in a manner which reveals that the cooperating terror outfits may conduct more, less or the same number of attacks as in the absence of cooperation; based on whether they are resource-constrained or not a priori; and on the extent to which cooperation can serve to ease such a constraint through inter-outfit resource-transfer. In the absence of external sponsorship, the paper shows that strategic cooperation between two outfits has no impact on terror activity if neither outfit is resource-constrained a priori. If only one outfit is resource-constrained a priori, on the other hand, then inter-group cooperation increases terror activity if and only if there is sufficient resource-asymmetry between the outfits. Further, if both outfits are resource-constrained a priori, then cooperation may increase or decrease terror activity depending on parametric asymmetries. Finally, it is demonstrated that while cooperation can neutralize the impact of strategic external sponsorship on terror activity and thereby remove the incentive for its provision, minor modifications to the sponsorship mechanism can often mitigate this phenomenon.

Keywords Terror outfit \cdot Terror attacks \cdot Non-cooperative competition \cdot Outfit cooperation \cdot External sponsorship \cdot Counter-terrorism

JEL Classification C71 · C72 · D74 · H79

1 Introduction

Terrorists perpetrate violence to draw public attention to their objectives, and to pressurize ruling political dispensations into capitulating to their demands. Just as governments of different countries may coalesce to combat terrorism, terrorist

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groups may join forces to overwhelm the State machinery.¹ For instance, consider the merger in 2012 of the Somali terrorist group al-Shabaab, with the al Qaeda.² Alliances between terrorist groups however, are an exception rather than the rule, given that less than one percent (417 to be exact) of the 81,799 terror attacks conducted during 1970-2007 involved more than one terror outfit (Asal et al. 2016). This may be due to the inability of terror outfits, which are illegal organizations, to credibly overcome commitment issues in the absence of third-party enforcement (Bacon 2017).³ Further, a significant fraction of outfits does not exist for more than a year, thereby making it difficult for them to reliably pledge to certain behavioral patterns for the long term.⁴

Ackerman et al. (2017) explore the circumstances under which terror outfits with differing ideologies may align operationally, to achieve common goals. The game-theoretic framework used by the authors for this purpose gives rise to multiple equilibria, with some characterized by cooperation. In fact, a prominent reason proposed in the literature for inter-outfit cooperation, is the resultant enhancement of outfit longevity. Using data spanning 1987 to 2005, Phillips (2014) shows that terror outfits having one ally are 38 percent less likely to discontinue in a given year, compared to terror outfits without any ally. Further, the abilities of terror outfits to address each other's organizational voids, forge a common discernibility and cultivate mutual trust are ubiquitous prerequisites for intergroup alliances (Bacon 2018a). The notion that alliances are a measure of vulnerability, however, is not empirically validated.⁵ On the other hand, Phillips (2019) finds that "alliances are associated with territorial control, intermediate membership size, and religious motivation".

In addition to understanding the causes of inter-group terrorist cooperation, it is also important to dwell on the nature of cooperation between terror outfits. Significant variation is observed in the scope and depth of cooperation between different terror outfits, from mergers and strategic cooperation at the upper end of the scale, to tactical and transactional cooperation at the lower end (Moghadam 2015). In fact, mergers and strategic cooperation become equivalent if payoffs are freely transferable between the outfits, under the latter regime. When outfits merge, each outfit sacrifices its individual identity. Under transactional cooperation, at the other end of the spectrum, there is usually no noteworthy loss of independence for either outfit. Hence, the quality of cooperation holds salience for each outfit, and thereby for those seeking to counter them.

¹ See Sandler (2005) for a discussion on coordination problems which plague international cooperation against transnational terrorism, but do not hinder resolute effort against domestic terrorism; and Perliger and Milton (2018) for a data-driven identification of conditions under which countries may engage in counter-terrorism cooperation.

 $^{^2}$ See Thomas (2013) for a discussion on the counter-terrorism opportunities arising from vulnerabilities created as a result of this amalgamation.

³ See Choi et al. (2016) for an insightful discussion on inter-group and intra-group dynamics, and possible feedback effects of inter-outfit rivalries. These can potentially negate any attempts at cooperation.

⁴ Phillips (2019), based on eight most extensive global datasets on the longevity of terror outfits, obtains that 25–74 percent of outfits do not last beyond a year.

⁵ See Phillips (2019), for instance.

The present work is the first to formally model inter-outfit strategic cooperation in a manner which reveals that the cooperating outfits may conduct more, less or the same number of attacks as in the absence of cooperation; based on whether they are resource-constrained or not a priori; and on the extent to which cooperation can serve to ease such a constraint through inter-outfit resource-transfer. The alleged provision of training facilities by the Hezbollah in southern Lebanon, for thousands of Hamas fighters, is a case in point.⁶ Bacon (2018b) discusses how cooperation between the al Qaeda and the Taliban, provided the former with a safe haven in Afghanistan, while benefitting the latter in terms of superior training of its fighters by al Qaeda operatives. She points out that al Qaeda operatives have, in fact, been known to carry out special operations on Taliban's behalf. Bacon (2018b) also mentions how it was the al Qaeda, during the 1990s, which provided funds to the Taliban. This typifies successful cooperation spanning over two decades, in which resources have been transferred in both directions during different periods of time, based on changing circumstances and evolving requirements. Also consider the alliance with the Popular Front for the Liberation of Palestine (PFLP), initiated by Fusako Shigenobu of the Japanese Red Army, in 1971. The cooperation, driven by resource requirements needed to implement its chosen strategy, resulted in the provision of guerilla training facilities to Red Army members, by PFLP operatives in Lebanon (Steinhoff 1976; Bacon 2018a).

Based on Bhan and Kabiraj (2020), our structure is able to illustrate clearly the distinction—if present—between the equilibria in the presence and absence of strategic cooperation, under different parametric restrictions. Further, the formulation demonstrates a natural barrier to the excessive use of any outfit channel for conducting attacks under cooperation, based on the diseconomies of scale associated with terror activity. This shows why such cost-convexities, by themselves, may provide a strong rationale for inter-outfit cooperation by providing the co-operating outfits multiple channels of terror activity.

Other benefits from strategic cooperation may flow from the internalization of operational externalities imposed by the activities of one group on the other, such as those discussed and modeled in Bhan and Kabiraj (2019). As a consequence of such cooperation, the total number of attacks conducted by the terrorists would tend to increase under positive externalities, and decrease under negative externalities. The present analysis, on the other hand, rationalizes strategic cooperation even in the absence of externalities, thereby indicating the possibility of inter-outfit cooperation in a wider range of real-world situations.

Refer to the afore-mentioned example of cooperation between the Japanese Red Army and the PFLP, the former originating in the East Asian country of Japan, and the latter operating in West Asia. Despite the traditional theatres of operation of these outfits being separated by thousands of kilometers of land and sea, their alliance led to the deadly attack conducted by Red Army terrorists on Lod Airport near

⁶ See "Israel says Hamas working with Hezbollah to train 'thousands' in Lebanon', in Times of Israel (9 June, 2018), https://www.timesofisrael.com/israel-says-hamas-working-with-hezbollah-to-train-thous ands-in-lebanon/.

the Israeli city of Tel Aviv in 1972, resulting in 28 deaths (including two attackers) and nearly 80 injuries (including the third attacker), thereby highlighting the potential for deadly cooperation between outfits imposing no operational externalities on each other a priori.

Inter-outfit cooperation may also have grave consequences in terms of the lethality of terror outfits. For instance, consider the symbiotic relationship that emerged between the Southeast Asian outfit Jemaah Islamiyah and the al Qaeda, which enabled the training of the former's manpower by the latter's operatives, resulting in the deadly Bali bombing in 2002 (Horowitz and Potter 2014). Also, the then alleged and oft-ridiculed—and later proven—training of amateur Boko Haram personnel by al Qaeda in the Islamic Maghreb (AQIM) operatives beginning in 2009, resulted in suicide attacks conducted by the former in 2011 on the United Nations office in Abuja, Nigeria, using tactics similar to bombings conducted by the latter (Aronson 2014). These examples serve to illustrate how cooperation can serve to increase the killing capacity of the outfits involved.

Finally, the circumstances associated with cooperation between symmetric and asymmetric entities, is critical in obtaining a holistic understanding of inter-group terrorist cooperation. Utilizing the UCDP/PRIO Armed Conflict Dataset, Bapat and Bond (2012) conclude that whereas outfits less at risk of State suppression tend to favour two-sided alliances, "vulnerable militants are more likely to form asymmetric alliances" such as those involving state or external sponsors. The present paper borrows from the formulation of Bhan and Kabiraj (2020) to illustrate not only the potential of strategic external sponsorship to augment violence, but also to demonstrate how strategic intergroup cooperation between terrorists can impede the effectiveness of such sponsorship, thereby decreasing the appeal for any potential sponsor to finance the cooperating outfits. This also provides a logical basis for a potential external sponsor, to hinder any inter-outfit strategic cooperation, in order to increase its own ability to induce additional terror attacks.

Consider for instance, the impact of the emergence of al-Badr in the Indian State of Jammu and Kashmir, towards the close of the 20th century. Earlier operating under the banner of Hizb-ul-Mujahideen (HM), Al-Badr was allegedly encouraged by Pakistan's Inter-Services Intelligence (ISI) to operate independently in the year 1998, as mentioned in an ANI report (dated 23 August, 2017) titled '*J-K: Al-Badr terrorist killed in Budgam encounter*'.⁷ Since then, the combined number of terror strikes conducted by both outfits dramatically increased, although HM still accounted for an overwhelming majority of the attacks. From 0 incidents in 1996 and 1997, the combined number of terror strikes jumped to 8 in 1999, 12 in 2000, and 11 in 2001. It is also noteworthy that Al-Badr was involved in only 1 terror incident (in 1999) out of the combined 31 in the period 1999-2001 (Global Terrorism Database). Hence, by engineering a split between HM and Al-Badr, the ISI was able to manipulate the former into conducting more attacks in order to maintain its (the HM's) pre-eminence.

⁷ See https://www.aninews.in/news/national/politics/j-k-al-badr-terrorist-killed-in-budgam-encounter/.

In the present paper we show that depending on the resources available with the outfits, their intrinsic propensities for violence and cost-efficiency parameters, cooperation may or may not increase the total number of attacks. Also, there are situations when cooperation reduces the total number of attacks. Further, the present work provides a theoretical foundation for strategic external sponsorship by internalizing the decision of terror outfits to cooperate strategically or not, and the external finance offered. Based on the *ex-ante* resources with the outfits and the quantum of finance made available by the sponsor, situations are illustrated where strategic external sponsorship can optimally induce outfits to operate non-cooperatively, and conduct attacks at the behest of the strategic sponsor.

Counter-terrorism (CT) implications of inter-group strategic cooperation must be viewed in light of the specificities of each instance in terms of *ex ante* resources with the outfits, availability of external sponsorship, etc., in order to determine whether such cooperation would increase or decrease terror strikes. Circumstances encouraging cooperation must be created in the latter situation, while measures inhibiting cooperation must be pursued in the former. For example, if the presence of a potential external sponsor is likely to increase attacks by discouraging cooperation, then CT efforts must be directed at enabling and encouraging alliance-formation, and thereby keeping the external sponsor at bay. Consider conversely, for instance, that cooperation is likely to ease the resource-constraint of an outfit such that overall violence is augmented. Then all efforts must be made to disrupt such an alliance by sowing distrust between the outfit leaders by raising suspicions of the potential partner being infiltrated by enemy intelligence, emphasizing ideological distinctions and operational autonomy, etc. via surveillance of inter-group communications and covert messaging for example, along the lines suggested by Bacon (2017).

The next section presents the baseline model, utilizing it to characterize and compare the equilibria under cooperation and non-cooperation. The third section analyzes the impact of strategic cooperation in the presence of a potential external sponsor. The fourth section extends the analysis by endogenizing the outfits' decision to cooperate or not, in the presence of strategic external sponsorship. Finally, the fifth section briefly discusses the implications of the results obtained, and concludes.

2 Model

Consider the interaction of two terror outfits, T_1 and T_2 , operating in a target country. We assume that initially, each outfit T_i (i = 1, 2) possesses some resources $R_i (> 0)$ of which a part is spent on terror activities and the remaining part on other non-terror activities, called consumption.⁸ Hence, the utility or payoff of T_i comes from two sources: consumption (X_i), and the attacks (A_i) it conducts.⁹

⁸ This may include expenditure on housing, health, education, etc. of the families of the members of the terror groups.

⁹ More generally, A_i can be considered as an index of terror activity. So A_i is assumed to be a continuous variable.

Assume the utility function to be linear, specifically,¹⁰

$$U_i = X_i + \alpha_i A_i; \quad i = 1, 2;$$

where the parameter $\alpha_i \geq 0$ represents intrinsic propensity for violence of T_i . The associated cost of conducting A_i attacks for T_i is

$$C_i(A_i) = \frac{1}{2}\beta_i A_i^2$$

where $\beta_i (> 0)$ is a parameter representing cost-efficiency of T_i , such that a higher β_i represents a lower efficiency. The quadratic cost function reflects increasing difficulty in conducting successive attacks. Then, the budget constraint of T_i is given by:

$$X_i + \frac{1}{2}\beta_i A_i^2 = R_i$$

We first note the equilibrium outcomes when the outfits interact independently or non-cooperatively, that is, when each outfit maximizes its payoff subject to its budget constraint. Following Bhan and Kabiraj (2020), we obtain the following outcomes:

2.1 Non-cooperative (NC) Equilibrium Outcomes

(*NC1*) When $R_i \ge \frac{1}{2}\beta_i \left(\frac{\alpha_i}{\beta_i}\right)^2$ holds for each *i*, that is, no outfit is resource-constrained, we call this interior equilibrium. Then in equilibrium:

$$A_i^{NC} = \frac{\alpha_i}{\beta_i}, \text{ and } X_i = R_i - \frac{1}{2} \frac{\alpha_i^2}{\beta_i} \equiv X_i^{NC} \ge 0 \quad \forall i = 1, 2$$
(1a)

Hence, total number of attacks is:

$$A^{NC} = A_1^{NC} + A_2^{NC} = \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}$$
(1b)

(*NC2*) When $R_1 \ge \frac{1}{2}\beta_1 \left(\frac{\alpha_1}{\beta_1}\right)^2$ but $R_2 < \frac{1}{2}\beta_2 \left(\frac{\alpha_2}{\beta_2}\right)^2$, the equilibrium outcomes will be:

$$A_1^{NC} = \frac{\alpha_1}{\beta_1}, X_1^{NC} = R_1 - \frac{1}{2} \frac{\alpha_1^2}{\beta_1} \ge 0$$
, but $A_2^{NC} = \sqrt{\frac{2R_2}{\beta_2}} X_2^{NC} = 0$ (2a)

 $^{^{10}}$ The formulation is based on Bhan and Kabiraj (2020).

$$A^{NC} = A_1^{NC} + A_2^{NC} = \frac{\alpha_1}{\beta_1} + \sqrt{\frac{2R_2}{\beta_2}}$$
(2b)

(*NC3*) When $R_i < \frac{1}{2}\beta_i \left(\frac{\alpha_i}{\beta_i}\right)^2$ holds $\forall i = 1, 2$, in equilibrium we have:

$$A_i^{NC} = \sqrt{\frac{2R_i}{\beta_i}} \text{ and } X_i^{NC} = 0 \quad \forall i = 1, 2$$
(3a)

$$A^{NC} = A_1^{NC} + A_2^{NC} = \sqrt{\frac{2R_1}{\beta_1}} + \sqrt{\frac{2R_2}{\beta_2}}$$
(3b)

We call the equilibria in (NC2) and (NC3) corner solutions—these are cases where at least one outfit is resource-constrained. Given the above equilibria, we shall now study whether the outfits collectively enhance terror activity under cooperation.

2.2 Co-operation Between Terror Outfits

We assume that under cooperation, payoffs are freely transferable between the outfits. This means that under cooperation, the outfits are concerned with the maximization of the sum of their payoffs, subject to the overall resource-constraint. Hence, strategic cooperation is equivalent to a merger of the outfits. The outfits will cooperatively decide the numbers of attacks to be conducted through each of the two outfit channels. After this allocation, all remaining resources will be consumed by the outfits. Note that the channel of consumption is irrelevant when maximizing joint utility.

As far as the incentive for cooperation is concerned, in the present paper there is no problem of coordination or externalities, nor is there any increase in cost-efficiency though cooperation.¹¹ Hence, the joint payoff under cooperation always being at least as large as the sum of their non-cooperative payoffs, explains the incentive for cooperation. Moreover, if the ultimate objective of the terrorists is to overpower the targeted country and take control, then the outfits are likely to attempt increasing the total number of terror strikes. We identify situations where the total number of attacks increases under cooperation, and try to derive insights into the problem.

Denoting $X_1 + X_2 = X$, the optimization problem under cooperation is

$$Max_{X,A_1,A_2}(U_1 + U_2) = X + \alpha_1 A_1 + \alpha_2 A_2$$

subject to the following constraints:

¹¹ Note that existence of coordination problems will tilt the choice towards non-cooperation, whereas the existence of synergies will favor cooperation.

Budget constraint: $R_1 + R_2 = X + \frac{1}{2} (\beta_1 A_1^2 + \beta_2 A_2^2)$ Non - negativity constraints: $X \ge 0$, $A_1 \ge 0$ and $A_2 \ge 0$.

Then the optimization problem can be restated as:

$$\max_{\left\{X,A_1,A_2,\lambda,\mu,\gamma_1,\gamma_2\right\}}L$$

where L is the Lagrangian, the expression for which is

$$L = X + \alpha_1 A_1 + \alpha_2 A_2 + \lambda \left[R_1 + R_2 - X - \frac{1}{2} \left(\beta_1 A_1^2 + \beta_2 A_2^2 \right) \right] + \mu X + \gamma_1 A_1 + \gamma_2 A_2$$

By solving the Kuhn-Tucker conditions to the above problem, we shall get the

following characterization of equilibrium under cooperation (C) (see "Appendix 1"): (C1) If $R_1 + R_2 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$, the cooperative equilibrium outcome is

$$A_i^C = \frac{\alpha_i}{\beta_i}, \quad i = 1, 2, \text{ and } \quad X^C = R_1 + R_2 - \frac{1}{2} \left(\frac{\alpha_1^2}{\beta_1} + \frac{\alpha_2^2}{\beta_2} \right) \ge 0$$
 (4a)

Then total number of attacks under this situation is

$$A^{C} = A_{1}^{C} + A_{2}^{C} = \frac{\alpha_{1}}{\beta_{1}} + \frac{\alpha_{2}}{\beta_{2}}$$
(4b)

(C2): If $R_1 + R_2 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$, the cooperative equilibrium outcome is:

$$A_i^C = \sqrt{\frac{2(R_i + R_j)\beta_i\beta_j}{\alpha_i^2\beta_j + \alpha_j^2\beta_i}} \left(\frac{\alpha_i}{\beta_i}\right), i \neq j \quad \text{and} \quad X^C = 0$$
(5a)

$$A^{C} = A_{1}^{C} + A_{2}^{C} = \sqrt{\frac{2(R_{1} + R_{2})\beta_{1}\beta_{2}}{\alpha_{1}^{2}\beta_{2} + \alpha_{2}^{2}\beta_{1}}} \left[\frac{\alpha_{1}}{\beta_{1}} + \frac{\alpha_{2}}{\beta_{2}}\right]$$
(5b)

2.3 Cooperative Versus Non-cooperative Outcomes

We can now examine whether under cooperation, the total number of attacks will increase compared to non-cooperation. We study this issue under four possible assumptions.

Assumption (A1)
$$R_1 + R_2 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$$
 along with $R_1 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1}$ and $R_2 \ge \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$

Given assumption (A1), under non-cooperative equilibrium none of the outfits are resource-constrained, and hence the equilibrium outcome is given by (NC1). The

corresponding equilibrium under cooperation is given by (C1). Then comparing (1) and (4) we have:

 $A_{i}^{C} = A_{i}^{NC}; i = 1, 2, \text{ and } A^{C} = A^{NC}$

Therefore, when none of the outfits are resource-constrained, cooperation will have no effect on the number of attacks.

Proposition 1 When neither outfit is resource-constrained, cooperation will have no impact on terror activity.

Assumption (A2) $R_1 + R_2 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$ along with $R_1 > \frac{1}{2} \frac{\alpha_1^2}{\beta_1}$ and $R_2 < \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$

Under this assumption, the equilibrium under non-cooperation is given by (NC2). This is the scenario when only one outfit (here T_2 is resource-constrained under competition, but the outfit cooperation does not face any resource-constraint. Hence, the cooperative equilibrium is once again given by (C1). So, to see the effect of cooperation on the number of attacks, we compare (2) and (4). We have the results:

$$A_1^C = A_1^{NC}, \ A_2^C > A_2^{NC} \text{ and } A^C > A^{NC}$$

The inequality in the second term arises because $R_2 < \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$. Thus, when only one outfit is resource-constrained under non-cooperation, at least some surplus resource from the resource-rich outfit (here T_1) is funneled to conduct more attacks through the resource-constrained outfit channel (T_2). Hence, the total number of attacks increases under cooperation.

Proposition 2 When only one outfit is resource-constrained while the other outfit has sufficiently large resources, cooperation enhances terror activity.

Assumption (A3)
$$R_1 + R_2 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2} along with R_1 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1} and R_2 < \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$$

Consider assumption (A3). This is the scenario when under non-cooperation, outfit T_2 is resource-constrained while T_1 is not. Moreover, the merged outfit faces a resource-constraint, meaning that it cannot conduct as many attacks it wants. Hence, non-cooperative equilibrium is given by (NC2) while the cooperative equilibrium is given by (C2). Then comparing (2) and (5) we have the following results: First, since under this scenario, $\sqrt{\frac{2(R_1+R_2)\beta_1\beta_2}{a_1^2\beta_2+a_2^2\beta_1}} < 1$, so we must have $A_1^C < A_1^{NC}$, that is, the number of attacks through the unconstrained outfit channel (T_1) falls under cooperation. Further, under the given conditions, we get $A_2^C > A_2^{NC}$. This follows from the fact that

$$\sqrt{\frac{2(R_1 + R_2)\beta_1\beta_2}{\alpha_1^2\beta_2 + \alpha_2^2\beta_1}} \frac{\alpha_2}{\beta_2} > \sqrt{\frac{2R_2}{\beta_2}} \Leftrightarrow R_1\alpha_2^2\beta_1 > R_2\alpha_1^2\beta_2 \Leftrightarrow \frac{R_1}{\frac{1}{2}\frac{\alpha_1^2}{\beta_1}} > \frac{R_2}{\frac{1}{2}\frac{\alpha_2^2}{\beta_2}}$$

which holds, given (A3). Finally, total number of attacks will go up (i.e., $A^C > A^{NC}$) if and only if the following holds, that is,

$$\sqrt{\frac{2(R_1 + R_2)\beta_1\beta_2}{\alpha_1^2\beta_2 + \alpha_2^2\beta_1}} \left[\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}\right] > \frac{\alpha_1}{\beta_1} + \sqrt{\frac{2R_2}{\beta_2}}$$
(6)

We can therefore write the following result:

Proposition 3 Under assumption (A3), cooperation between the two outfits enhances the total number of attacks if and only if the outfit which is resource-constrained a priori, is sufficiently small compared to the other outfit in terms of resources.

Proof We prove the result in a special case, when both the outfits are equally efficient at conducing attacks, and have the same intrinsic propensity for violence. Suppose $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$. Then the condition (6) reduces to

$$2\sqrt{\frac{\left(R_{1}+R_{2}\right)}{\beta}} > \frac{\alpha}{\beta} + \sqrt{\frac{2R_{2}}{\beta}} \tag{7}$$

Then there always exists (R_1, R_2) satisfying $R_1 + R_2 < \frac{\alpha^2}{\beta}$ and $R_1 \ge \frac{1}{2} \frac{\alpha^2}{\beta} > R_2$ such that the above inequality holds.¹² This proves the result. QED

Proposition 3 must be understood in the context of transferring resources from the resource-abundant outfit (or channel of attack) to the resource-constrained outfit. In the vicinity of the initial equilibrium, this would leave the former's attacks unchanged, while easing the latter's resource-constraint and thereby enabling it to optimally conduct additional attacks. This would lead to higher overall attacks in the vicinity of the initial equilibrium. Further resource-transfer in the same direction, however, is optimal under cooperation, as demonstrated earlier.¹³ Beyond a point, such a transfer would cause the former outfit's resource-constraint to bind, thereby causing its attacks to decline. However, this would be more (less) than proportion-ately compensated by the increase in the latter outfit's attacks, if and only if the latter outfit is sufficiently (insufficiently) small compared to the former, because of diseconomies in conducting attacks driven by the convex cost functions.

Assumption (A4)
$$R_1 + R_2 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2} along with R_1 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1} and R_2 < \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$$

Finally, consider assumption (A4). This is the scenario when not only is the outfit cooperation as a whole resource-constrained, but both outfits are also individually resource-constrained a priori. Therefore, the non-cooperative equilibrium is given

¹² We can simply fix R1+R2, then increase R1 and decrease R2 to satisfy the inequality (7).

¹³ Refer to the resource-allocation derived earlier, under the cooperative equilibrium given by (C2).

by (NC3), and the cooperative equilibrium by (C2). Hence, comparing equations (3) and (5), we can see that

$$A_i^C > A_i^{NC} \quad \text{iff} \quad R_j \alpha_i^2 \beta_j > R_i \alpha_j^2 \beta_i \quad i \neq j$$
(8)

and hence

$$A^{C} > A^{NC} \quad \text{iff} \quad \sqrt{\frac{2(R_{1} + R_{2})\beta_{1}\beta_{2}}{\alpha_{1}^{2}\beta_{2} + \alpha_{2}^{2}\beta_{1}}} \left[\frac{\alpha_{1}}{\beta_{1}} + \frac{\alpha_{2}}{\beta_{2}}\right] > \sqrt{\frac{2R_{1}}{\beta_{1}}} + \sqrt{\frac{2R_{2}}{\beta_{2}}} \tag{9}$$

Given the parametric restrictions under this case, the inequalities (8) and (9) may or may not hold, meaning that inter-outfit cooperation may increase or decrease the number of attacks by each outfit channel as well as the total number of attacks. We check the results in the following special cases:

Case (i) $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$ and $R_1 = R_2$. We expectedly obtain $A_i^C = A_i^{NC} \forall i = 1, 2$, and $A^C = A^{NC}$, that is, if the outfits are identical in respect of all parameters, cooperation will have no effect. Since both the outfits are identical in every respect, there is nothing additional to share under cooperation.

Case (ii) $\alpha_1 = \alpha_2, \beta_1 = \beta_2$ but $R_1 \neq R_2$. Here, we get $A^C > A^{NC}$.¹⁴ Without any loss of generality, suppose $R_1 > R_2$. Then $A_1^C < A_1^{NC}$ and $A_2^C > A_2^{NC}$. So when the outfits differ only in respect of the size of their resources, cooperation will lead to a higher number of total attacks, such that the number of attacks through the outfit channel which has lesser resources will increase. The result is intuitive. Since $R_1 > R_2$, therefore under non-cooperation, $A_1^{NC} > A_2^{NC}$. Now given that the cost of conducting attacks is increasing and convex, the marginal cost of attacking through T_1 under non-cooperative competition is larger than that through T_2 . So under cooperation, reallocation of resources from channel T_1 to channel T_2 will be mutually rewarding, that is, A_1 will fall and A_2 will rise. Reducing A_1 by one unit will release resources for increasing A_2 by more than one unit. Therefore, the total number of attacks (A) will increase.

Case (iii) $\beta_1 = \beta_2$, $R_1 = R_2$ but $\alpha_1 \neq \alpha_2$. Here, we obtain $A^C < A^{NC}$.¹⁵ If $\alpha_1 > \alpha_2$, then we have $A_1^C > A_1^{NC}$ and $A_2^C < A_2^{NC}$, that is, the number of attacks through the outfit channel having a higher intrinsic propensity for violence increases while that through the other channel falls. The total number of attacks also falls, given that the outfits differ in respect of their violence propensities. The intuition of this result also hinges on cost-convexities. Because the attacks conducted by each outfit in the

¹⁴ Under Case (ii), $A^C = 2\sqrt{\frac{(R_1+R_2)}{\beta}}$ and $A^{NC} = \sqrt{\frac{2R_1}{\beta}} + \sqrt{\frac{2R_2}{\beta}}$. Therefore, $A^C > A^N$ because $\frac{(R_1+R_2)}{2} > \sqrt{R_1R_2}$, that is, A.M. > G.M, where the abbreviations refer to the arithmetic and geometric means of R_1 and R_2 respectively.

¹⁵ Under Case (iii),
$$A^{C} = 2\sqrt{\frac{2R}{\beta}} \frac{(\alpha_{1} + \alpha_{2})}{\sqrt{2(\alpha_{1}^{2} + \alpha_{2}^{2})}}$$
 and $A^{NC} = 2\sqrt{\frac{2R}{\beta}}$. Hence, $A^{C} < A^{NC}$ because $\frac{(\alpha_{1} + \alpha_{2})}{\sqrt{2(\alpha_{1}^{2} + \alpha_{2}^{2})}} < 1$.

non-cooperative equilibrium are equal and independent of the intrinsic propensity for violence, resource-reallocation from one outfit to the other leads to efficiency loss at the margin, due to the increasing and strictly convex cost of conducting attacks. But given $\alpha_1 > \alpha_2$, since resources are drawn from outfit channel T_2 to conduct additional attack through T_1 , payoff of the outfit cooperation will increase at the margin. This explains why the number of attacks through T_1 increases, while that through T_2 falls. But given the strictly convex cost function, the fall of attacks in equilibrium must dominate the increase, thereby leading to a lower total number of attacks under cooperation.

Case (iv) $\alpha_1 = \alpha_2$, $R_1 = R_2$ but $\beta_1 \neq \beta_2$. Here we unambiguously obtain $A^C > A^{NC}$, that is, cooperation will enhance the total number of attacks.¹⁶ Without any loss of generality when $\beta_1 > \beta_2$, we get $A_1^C < A_1^{NC}$ and $A_2^C > A_2^{NC}$, implying that the inefficient outfit channel will conduct less attacks under cooperation. Since more and more attacks are conducted through efficient channel, the total number of attacks will go up.

Therefore, given assumption (A4), we arrive at the following proposition:

Proposition 4 When both outfits are resource-constrained a priori and the outfits differ in respect of at least one parameter, cooperation will affect the number of attacks to be conducted by each outfit as well as the total number of attacks. In particular, if the outfits have different levels of resources or if they differ in respect of their efficiency in conducting attacks, the total number of attacks under cooperation must increase. On the other hand, if the outfits have different intrinsic propensities of violence, cooperation will reduce the total number of attacks.

To summarize the results of this section, we have shown that the effect of cooperation on terror activity depends on available resources, intrinsic propensities for violence and cost-efficiency parameters of the outfits. Cooperation will increase the total number of attacks under assumption (A2), under assumption (A3) if condition (6) holds, and under assumption (A4) if condition (9) holds. However, cooperation may sometimes also reduce the total number of attacks (see assumption (A3) when condition (6) does not hold, and assumption (A4) if the inequality in condition (9) is reversed). Under assumption (A1), however, cooperation has no effect.

Finally, as far as the choice between cooperation and non-cooperation is concerned, since in our paper we have assumed that the outfit cooperation maximizes the sum of utility of the outfits and that payoffs are transferable between the outfits, cooperation will weakly dominate non-cooperation from the perspective of the outfits. When the cooperative and non-cooperative outcomes (i.e., terror activities)

¹⁶ Here $A^C = \sqrt{\frac{2R}{\beta_1\beta_2}}\sqrt{2(\beta_1 + \beta_2)}$ and $A^{NC} = \sqrt{\frac{2R}{\beta_1\beta_2}}\left(\sqrt{\beta_1} + \sqrt{\beta_2}\right)$, hence $A^C > A^{NC}$ because A.M. > G.M., where the abbreviations refer to the arithmetic and geometric means of β_1 and β_2 respectively.

are the same (for example, this is the case under assumption (A1)), the outfits will be indifferent about its choice (given that there are no coordination or externalities problems). However, when the cooperative and non-cooperative outcomes are different, cooperation will be strictly preferred to non-cooperation. In the following section we now introduce the possibility of external sponsorship and study the consequences.

3 Cooperation Under Sponsorship

There is ample evidence of terror outfits receiving funds from different agencies such as charities and NGOs.¹⁷ A part of this sponsorship is provided strategically, to induce more attacks.

Consider the availability of external sponsorship F > 0 (measured in units of consumption). Further, assume that the sponsor commits to distribute this fund *ex post* between the outfits, in proportion to the number of terror attacks conducted by each.¹⁸ Thus, T_i . receives $F_i = \frac{A_i}{A_i + A_j}F$. In the presence of such sponsorship, the payoff function of T_i (i = 1, 2), becomes

$$U_i = X_i + \alpha_i A_i + F_i \tag{10}$$

After incorporating the budget constraint, the payoff maximization problem of T_i (i = 1, 2) under non-cooperation becomes

$$Max_{A_{i}}U_{i} = R_{i} - \frac{1}{2}\beta_{i}A_{i}^{2} + \alpha_{i}A_{i} + F_{i}.$$
(11)

Bhan and Kabiraj (2020) have shown that the equilibrium solution to the above problem is stable and unique, and we denote this by (A_1^*, A_2^*) . brief outline of the solution is provided in "Appendix 2". It is shown that when resources are sufficiently large (i.e., $R_i > \frac{1}{2} \frac{a_i^2}{\beta_i}$; i = 1, 2), the reaction functions are initially upward sloping, intersect the 45°-line, and then slope downwards. In this case, given that each outfit T_i (i = 1, 2) is competing for larger share of external sponsorship, T_i will conduct more than $\frac{a_i}{\beta_i}$ attacks. This illustrates the possibility that external sponsorship can induce more attacks when the outfits compete non-cooperatively. In fact, if the outfits play non-cooperatively and not all outfits are resource-constrained initially, the total number of attacks will increase under external sponsorship.

Now suppose that given the commitment of the sponsor, the outfits decide to act cooperatively and hence maximize the sum of their payoffs. Hence the problem is:

$$Max_{X_1,X_2,A_1,A_2}(U_1 + U_2) = X_1 + X_2 + \alpha_1 A_1 + \alpha_2 A_2 + F$$
(12)

¹⁷ See Chadha (2015) for a comprehensive discussion on the sources of terror finance, and also the discussion in Bhan and Kabiraj (2020).

¹⁸ This is called the proportionate external sponsorship rule or mechanism (Bhan and Kabiraj 2020).

subject to the budget constraint,

$$X_1 + X_2 + \frac{1}{2} \left(\beta_1 A_1^2 + \beta_2 A_2^2 \right) = R_1 + R_2$$

One can see that if $R_1 + R_2 \ge \frac{1}{2} \left(\frac{\alpha_1^2}{\beta_1} + \frac{\alpha_2^2}{\beta_2} \right)$, then an interior optimum exists. Otherwise, there is a corner solution. In either case, the solution to the above optimization problem is independent of *F*, and hence identical to the solution to the optimization problem of subsection 2.2 (absence of sponsorship). We therefore arrive at Proposition 5.

Proposition 5 If terror outfits play cooperatively, then the number of terror strikes conducted by each group in the presence of ex post proportionate external sponsorship will be identical to that in the absence of external sponsorship.

The intuition for this result rests on the fact that external sponsorship loses its ability to induce terror strikes because, irrespective of the values of A_1 and A_2 , the groups together would receive $F_1 + F_2 = F$. Hence, the number of terror strikes each outfit conducts will depend only on those factors which determine the equilibrium levels in the absence of external sponsorship, thereby ensuring a solution identical to that in the absence of external sponsorship. Then the following is a straight-forward corollary.

Corollary If terror outfits co-operate strategically, there is no incentive for providing ex post proportionate external sponsorship.

It seems intuitive that in an environment characterized by the presence of multiple terror outfits and a common potential external sponsor, greater strategic cooperation between the terror outfits would impede the ability of the sponsor to manipulate the behavior of the outfits. This, in turn, would weaken the incentive for the external sponsor to provide sponsorship. The sponsor would therefore have an incentive to hinder strategic co-operation or engineer a split between the terror outfits, in order to increase its own influence on their actions. This is allegedly what happened in the case of Hizb-ul-Mujahideen (HM) in 1998, as discussed earlier.

Finally, note that given the structure of the game, to the question of whether the outfits will decide their terror activities cooperatively or non-cooperatively, it follows from the mathematical formulation of the problem that cooperation will dominate non-cooperation from the perspective of the outfits.

4 Further Extension

In this section, we explore the circumstances in which an external sponsor would provide funds to induce increased terror attacks. Since the joint payoff of the outfits under cooperation is never less than the sum of their non-cooperative payoffs, the outfits may optimally decide to cooperate if possible, irrespective of whether any sponsorship (under the proportionate allocation rule) is available or not. Then, from

the corollary to Proposition 5, it follows that the scope for inducing increased terror activity by providing proportionate external sponsorship is limited since cooperation is never less beneficial than non-cooperation from the perspective of the outfits. Hence, our model thus far, fails to adequately rationalize *ex post* proportionate external sponsorship. In the analysis below, we slightly modify the structure of the game and restrict it to the assumption that external sponsorship is offered if and only if it increases the total number of attacks. Then it follows from the analysis of Sect. 3 that external sponsorship is offered only if the outfits behave non-cooperatively. Thus, the main idea of the present section is to show that the external sponsor can choose the sponsorship amount F to incentivize the outfits to behave non-cooperatively and increase the total number of attacks.

Suppose that initially, an external sponsor commits not to pay F > 0 unless the outfits play a non-cooperative game to determine the levels of their terror activities. In the following analysis, if the external sponsor offers any positive level of funding to the outfits, we shall call such a regime F. If no sponsorship is offered initially (i.e., F = 0) however, and then the outfits decide optimally whether to play the game cooperatively or non-cooperatively, we shall call this regime \emptyset . We have already noted in Sect. 2, that under this situation playing the game cooperatively will weakly dominate playing non-cooperatively. The reason is that the outfits are never worse off under cooperation compared to non-cooperation, that is, $U^C(\emptyset) \ge U^{NC}(\emptyset)$, where $U^{\tau} = U_1^{\tau} + U_2^{\tau}$, $\tau \in \{NC, C\}$. So, it may be presumed that the outfits under regime \emptyset will play the game cooperatively.¹⁹ Then F > 0 will be committed only if $A^{NC}(F) > A^C(\emptyset)$, that is, if the total number of terror attacks under F regime is larger than that under \emptyset regime. But such an offer will be rejected by the outfits unless $U^{NC}(F) \ge U^C(\emptyset)$, that is, the outfits are not worse off by accepting the F contract. We shall now discuss the problem under the various assumptions we have made in Sect. 2.2 (i.e., Assumptions (A1)–(A4)).

Assumption (A1) $R_1 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1}, R_2 \ge \frac{1}{2} \frac{\alpha_2^2}{\beta_2}, \text{ so } R_1 + R_2 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$

Sub-case (i) $R_j \ge \frac{1}{2} \frac{a_j^2}{\beta_j}$ and $R_1 + R_2 > \frac{1}{2} \frac{a_1^2}{\beta_1} + \frac{1}{2} \frac{a_2^2}{\beta_2}$. From Sect. 2.3, we have $A^C(\emptyset) = A^{NC}(\emptyset) = \frac{a_i}{\beta_i} + \frac{a_j}{\beta_j} < \sqrt{\frac{2R_i}{\beta_i}} + \sqrt{\frac{2R_2}{\beta_2}}$, i.e., when, while cooperation is no worse for the outfits than non-cooperation, the former does not increase the number of attacks. On the other hand, when F > 0 is offered, it will be accepted by the outfits because $U^{NC}(F) > U^C(\emptyset) (= U^{NC}(\emptyset))$, and given Assumption (A1), we must have $A^{NC}(F) > A^C(\emptyset)$, because $A_i^{NC}(F) > \frac{a_i}{\beta_i}$ but $A_j^{NC}(F) \ge \frac{a_j}{\beta_j}$. Therefore, under th-case, sponsorship will occur and the number of attacks will increase. Since the maximum number oattacks that T_1 and T_2 can conduct cannot exceed $\sqrt{\frac{2R_1}{\beta_1}}$ and $\sqrt{\frac{2R_2}{\beta_2}}$.

¹⁹ Note that the inferences pertaining to optimal external sponsorship, obtained in this section, remain unaffected even without this assumption.

²⁰ It must be borne in mind that in the absence of cooperation, the terror activity level under proportionate external sponsorship is never less than that in its absence. That is, $A^{NC}(F) \ge A^{NC}(\emptyset)$.

respectively, the sponsor can choose F. strategically such that the outfits conduct these many attacks.²¹

Sub-case (ii) $R_i = \frac{1}{2} \frac{\alpha_i^2}{\beta_i}$ for, and $R_1 + R_2 = \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$. In this case $A^C(\emptyset) = A^{NC}(\emptyset) = \frac{\alpha_i}{\beta_i} + \frac{\alpha_j}{\beta_j} = \sqrt{\frac{2R_i}{\beta_i}} + \sqrt{\frac{2R_2}{\beta_2}}$ since under sponsorship (F > 0) total number of attacks will be $A^{NC}(F) = A^C(\emptyset)$, therefore under this Sub-case, no sponsorship will be available.

Assumption (A2)
$$R_1 > \frac{1}{2} \frac{\alpha_1^2}{\beta_1}, R_2 < \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$$
 but $R_1 + R_2 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$.

From Sects. 2.1 and 2.2, we have $A_1^{NC}(\emptyset) = \frac{\alpha_1}{\beta_1}$, $A_2^{NC}(\emptyset) = \sqrt{\frac{2R_2}{\beta_2}} < \frac{\alpha_2}{\beta_2}$, $A_1^C(\emptyset) = \frac{\alpha_1}{\beta_1}$ and $A_2^C(\emptyset) = \frac{\alpha_2}{\beta_2}$, so that $A^C(\emptyset) > A^{NC}(\emptyset)$. We further have $U^C(\emptyset) > U^{NC}(\emptyset)$, that is, the outfits will be strictly better off choosing their terror activities cooperatively when no sponsorship is available, and under this situation it so happens that the number of attacks is higher than that under non-cooperation. Hence, under regime \emptyset , $A^C(\emptyset) = \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}$. Correspondingly, the joint profits of the outfits are,

$$U^{C}(\emptyset) = \alpha_{1}\left(\frac{\alpha_{1}}{\beta_{1}}\right) + \alpha_{2}\left(\frac{\alpha_{2}}{\beta_{2}}\right) + R_{1} + R_{2} - \frac{1}{2}\left(\frac{\alpha_{1}^{2}}{\beta_{1}} + \frac{\alpha_{2}^{2}}{\beta_{2}}\right) = \frac{1}{2}\left[\frac{\alpha_{1}^{2}}{\beta_{1}} + \frac{\alpha_{2}^{2}}{\beta_{2}}\right] + R_{1} + R_{2}$$
(13)

The question then remains whether by committing an appropriate amount of funds, conditional on the terror outfits playing the game non-cooperatively, the sponsor can induce the outfits to further increase the total number of attacks. We show that if R_1 is sufficiently large, the sponsor can appropriately choose an F > 0 to maximize the total number of attacks.

If any F > 0 is offered by the sponsor and accepted by the terror outfits, then given Assumption (A2), the optimal number of terror attacks chosen by T_2 will be $A_2^{NC}(R_2) = \sqrt{\frac{2R_2}{\beta_2}}$, and the optimal number of terror attacks to be chosen by T_1 will be

$$A_1^{NC}(F;R_1,R_2) = \min\{A_1(F;A_2^{NC}(R_2)), \sqrt{\frac{2R_1}{\beta_1}}\}$$
(14)

where $A_1(F;A_2^{NC}(R_2))$ is the solution obtained from the first order condition (FOC) of the problem: $Max_{A_1}U_1 = R_1 - \frac{1}{2}\beta_1A_1^2 + \alpha_1A_1 + F_1$, where $F_1 = \frac{A_1}{A_1 + A_2}F$ and $A_2 = A_2^{NC}(R_2)$. The FOC is:

²¹ As long as $A_i \leq \sqrt{\frac{2R_i}{\beta_i}}$, i = 1, 2, for any F > 0, A_i 's are solved from the first order conditions (FOCs) of the utility maximization problem under non-cooperative situation, i.e., $\alpha_i + \frac{A_j}{(A_i + A_j)^2}F - \beta_iA_i = 0$; i = 1, 2. Now setting $A_i = \sqrt{\frac{2R_i}{\beta_i}}$ for i = 1, 2 from the FOCs, we shall get the optimal level of sponsorship which maximizes the total number of attacks to be $F = (\beta_1 A_1 + \beta_2 A_2) - (\beta_1 + \beta_2)$.

$$\alpha_1 + \frac{A_2}{\left(A_1 + A_2\right)^2} F - \beta_1 A_1 = 0 \tag{15}$$

Given that the second order condition (SOC) is satisfied, $A_1(F;A_2^{NC}(R_2))$ is solved from the above. Now, as long as $A_1(F;A_2^{NC}(R_2)) < \sqrt{\frac{2R_1}{\beta_1}}$, F can be increased to raise $A_1^{NC}(.)$ to $\sqrt{\frac{2R_1}{\beta_1}}$. Hence, the optimal F maximizing the total number of attacks in this situation is given by $F^* = F(R_1;R_2)$, solved from $A_1^{NC}(F;R_1,R_2) = \sqrt{\frac{2R_1}{\beta_1}}$.²² Therefore, $F(R_1;R_2)$ will be offered by the sponsor provided the total number of terror attacks under F > 0 (non-cooperative competition) is larger than that under F = 0 (cooperative situation), i.e., $A^{NC}(F) > A^C(\emptyset)$, or, $\sqrt{\frac{2R_1}{\beta_1}} + \sqrt{\frac{2R_2}{\beta_2}} \ge \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}$. This can also be expressed as

$$R_1 > \frac{\beta_1}{2} \left[\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} - \sqrt{\frac{2R_2}{\beta_2}} \right]^2 \equiv R_1^*$$
(16)

So the sponsor will want to induce (A_1^{NC}, A_2^{NC}) terror attacks when condition (16) holds, and the optimal sponsorship $F(R_1; R_2)$ is obtained from the FOC $\alpha_1 + \frac{A_2}{(A_1+A_2)^2}F - \beta_1A_1 = 0$ as

$$F(R_1;R_2) = (\beta_1 A_1 - \alpha_1) \frac{(A_1 + A_2)^2}{A_2}$$
(17)

Finally, given $R_1 > R_1^*$ offer $F(R_1; R_2)$ will be acceptable to the outfits if and only if $U^{NC}(F) \ge U^C(\emptyset)$. We have

$$U^{NC}(F) = \alpha_1 (A_1^{NC}) + \alpha_2 (A_2^{NC}) + R_1 + R_2 - \frac{1}{2} \beta_1 (A_1^{NC})^2 - \frac{1}{2} \beta_2 (A_2^{NC})^2 + F$$

= $\alpha_1 \left(\sqrt{\frac{2R_1}{\beta_1}} \right) + \alpha_2 \left(\sqrt{\frac{2R_2}{\beta_2}} \right) + R_1 + R_2 - \frac{1}{2} \left[\beta_1 \left(\sqrt{\frac{2R_1}{\beta_1}} \right)^2 + \beta_2 \left(\sqrt{\frac{2R_2}{\beta_2}} \right)^2 + F(R_1; R_2) \right]$

On simplification,

$$U^{NC}(F) = \alpha_1 \left(\sqrt{\frac{2R_1}{\beta_1}} \right) + \alpha_2 \left(\sqrt{\frac{2R_2}{\beta_2}} \right) + F(R_1; R_2)$$
(18)

Therefore, $U^{NC}(F) \ge U^{C}(\emptyset)$ if and only if (comparing Eqs. (13) and (18)),

$$\alpha_1 \left[\left(\sqrt{\frac{2R_1}{\beta_1}} \right) - \frac{1}{2} \left(\frac{\alpha_1}{\beta_1} \right) \right] + F(R_1; R_2) \ge \alpha_2 \left[\frac{1}{2} \left(\frac{\alpha_2}{\beta_2} \right) - \sqrt{\frac{2R_2}{\beta_2}} \right]$$
(19)

The left-hand side of Eq. (19) is strictly positive, but the right-hand side can be positive or negative or zero. Hence, a sufficient condition to satisfy Eq. (19) is

²² For all $F \ge F(R_1, R_2)$, A_1^{NC} will remain fixed at $\sqrt{\frac{2R_1}{\beta_1}}$.

 $\frac{\alpha_2}{\beta_2} \le 2\sqrt{\frac{2R_2}{\beta_2}}$, that is, R_2 is sufficiently large. In general, condition (19) will be satisfied if R_1 is sufficiently large. Both conditions (16) and (19) must hold, for any F > 0 to be offered by the sponsor, and accepted by the outfits.

Assumption (A3) $R_1 \ge \frac{1}{2} \frac{\alpha_1^2}{\beta_1}; R_2 < \frac{1}{2} \left(\frac{\alpha_2^2}{\beta_2} \right), \text{ but } R_1 + R_2 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}.$

Sub-case (i) $R_1 > \frac{1}{2} \frac{\alpha_1^2}{\beta_1}$ and $R_2 < \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$ and $R_1 + R_2 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$ In this case $A^C(\emptyset) = \sqrt{\frac{2(R_1 + R_2)\beta_1\beta_2}{\alpha_1^2\beta_2 + \alpha_2^2\beta_1}} \left[\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}\right] < \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}$ because $\sqrt{\frac{2(R_1 + R_2)\beta_1\beta_2}{\alpha_1^2\beta_2 + \alpha_2^2\beta_1}} < 1$. The analysis in this case will be similar to the previous case. Here, however, we have limited flexibility to increase R_1 to satisfy a condition like Eq. (16).²³

Sub-case (ii) $R_1 = \frac{1}{2} \frac{\alpha_1^2}{\beta_1}$ and $R_2 < \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$, but $R_1 + R_2 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$ From Sects. 2.2 and 2.3 we know that in the absence of external sponsorship, the outfits will play cooperatively, and the total number of attacks under this situation will be $A^C(\emptyset) = \sqrt{\frac{2(R_1 + R_2)\beta_1\beta_2}{\alpha_1^2\beta_2 + \alpha_2^2\beta_1}} \left[\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}\right]$ where $\frac{\alpha_1}{\beta_1} = \sqrt{\frac{2R_1}{\beta_1}}$ and $\frac{\alpha_2}{\beta_2} > \sqrt{\frac{2R_2}{\beta_2}}$. Then, from Sect. 3 it follows that if F > 0 be offered by the sponsor, the total number of attacks

would be $A^{NC}(F) = A^{NC}(\emptyset) = \frac{\alpha_1}{\beta_1} + \sqrt{\frac{2R_2}{\beta_2}}$ under non-cooperation. Hence under the assumption of this Sub-case, sponsorship will be provided if and only if condition (6) holds with reverse inequality.

Assumption (A4)
$$R_1 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1}, R_2 < \frac{1}{2} \frac{\alpha_2^2}{\beta_2}, \text{ so } R_1 + R_2 < \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$$

Here, inter-outfit cooperation will occur under regime \emptyset , and the outfits will conduct a total of $A^{C}(\emptyset)$ attacks. Now if condition (9) holds so that $A^{C}(\emptyset) > A^{NC}(\emptyset)$, then for any F > 0 which induces non-cooperation, the outfits would together conduct $A^{NC}(F)$ attacks where $A^{NC}(F) = A^{NC}(\emptyset) < A^{C}(\emptyset)$. So no external sponsorship will be provided, since it is counterproductive from the perspective of the sponsor because it reduces terror activity.²⁴ On the other hand, if condition (9) holds with reverse inequality so that $A^{NC}(\emptyset) > A^{C}(\emptyset)$ there exists F > 0 such that $U^{NC}(F) \ge U^{C}(\emptyset)$ which would induce the outfits to play non-cooperatively and thereby conduct $A^{NC}(F) = \sqrt{\frac{2R_1}{\beta_1}} + \sqrt{\frac{2R_2}{\beta_2}}$ attacks, where $A^{NC}(F) = A^{NC}(\emptyset) > A^{C}(\emptyset)$

²³ Here, $A^{NC}(F) > A^{C}(\emptyset)$ iff $R_{1} > \frac{\beta_{1}}{2} \left[\sqrt{\frac{2(R_{1}+R_{2})\beta_{1}\beta_{2}}{a_{1}^{2}\beta_{2}+a_{2}^{2}\beta_{1}}} \left(\frac{\alpha_{1}}{\beta_{1}} + \frac{\alpha_{2}}{\beta_{2}} \right) - \sqrt{\frac{2R_{2}}{\beta_{2}}} \right]^{2}$. By implicitly solving this inequality, there exists some R_{1}^{**} such that $A^{NC}(F) > A^{C}(\emptyset)$ iff $R_{1} > R_{1}^{**}$. Since $\sqrt{\frac{2(R_{1}+R_{2})\beta_{1}\beta_{2}}{a_{1}^{2}\beta_{2}+a_{2}^{2}\beta_{1}}} < 1$, we must have $R_{1}^{**} < R_{1}^{*}$, where R_{1}^{*} is as defined in the discussion of Assumption (A2). Hence, the condition required for external sponsorship to be provided in the present case is $R_{1}^{**} < R_{1} < R_{1}^{*}$.

²⁴ In fact, no external sponsorship will be provided even if condition (9) holds with equality, because the number of attacks under sponsorship will be equal to that in its absence, i.e., $A^{NC}(F) = A^{NC}(\emptyset) = A^{C}(\emptyset)$.

Hence, given Assumption (A4), external sponsorship can induce additional terror strikes if and only if condition (9) holds with reverse inequality.

To summarize this section, there are scenarios under each of Assumptions (A1) through (A4) where the external sponsor can choose *ex post* proportionate sponsorship appropriately to induce higher terror activity. Of most interest appears to be Assumption (A4), where it may be possible for such sponsorship to induce higher terror activity despite both outfits being resource-constrained a priori.

So the amount of resources available to the outfits initially, plays a crucial role in our analysis. There are, in fact, other alternative sponsorship mechanisms observed in the real world. Such mechanisms may act both as an incentivizing device and as an enabler of terror activity. For instance, when the outfits are resource-constrained, some funds may be provided before the terror activity has taken place. Then such a fund will relax, at least to some extent, the resource-constraints of the outfits, thereby enabling them to conduct more attacks. Hence, we may write the following proposition:

Proposition 6 *There are circumstances where external sponsorship increases terror activity.*

5 Conclusion

In the present paper, we have shown that when terror outfits differ from each other in some aspect or the other, there are situations when the outfits may gain through cooperation via inter-outfit resource-reallocation and the consequent increase in the total number of attacks. Generally, a terror outfit prefers to work independently in order to preserve its identity and autonomy. But there is evidence of inter-outfit strategic cooperation in certain situations. When outfits are not too distant ideologically, for instance, they may be willing to coordinate their activities. Coordinated transfer of resources and terror technology can enable terror outfits to enhance the number of attacks, and thereby reap benefits via the exploitation of loopholes in the state's security apparatus.

We have shown that benefits of strategic cooperation accrue to the cooperating outfits, when at least one outfit is resource-constrained. Through cooperation, the outfits can reallocate resources to conduct attacks more efficiently, or in favor of the more aggressive outlet. Inter-outfit cooperation can also derive benefits from cost-convexities. However, if an external sponsor commits to provide funds to the outfits in proportion to their attacks, cooperation will reduce the total number of attacks compared to non-cooperation. Hence, no strategic external sponsor will commit any funds to the outfits in this scenario. We have, however, subsequently modified the game and demonstrated situations rationalizing the existence of external sponsorship.

This paper seeks to provide insights to policy makers, to enable better designing of counter-terrorism (CT) policies. Defensive CT policies generally increase the cost of terrorist operations. The present analysis also underscores the importance of preventing the transfer of resources from one terror outfit to another. To this end, offensive policies aimed at destroying terrorist infrastructure or confiscating resources, may appear effective. However, such a policy may sometimes be very expensive to implement, both in pecuniary and non-pecuniary terms. Confidence building measures, that target one or the other outfit to restore normalcy, may not be very effective in view of the possible funneling of resources from one outfit to the other.

Finally, and more generally, our analysis demonstrates that inter-outfit strategic cooperation can serve to increase terror attacks under certain circumstances, while serving to inhibit terror activity under other situations. An example of the former is when a resource-constrained outfit cooperates with a resource-abundant outfit having sufficiently large resources, in the absence of external funding. On the other hand, we have discussed multiple situations where external sponsorship can be offered strategically to enhance terror activity by inhibiting inter-group cooperation. Hence, CT efforts targeted at disrupting cooperation under the former set of circumstances, while those aimed at curbing the leverage of the external sponsor over the terrorists by encouraging intergroup cooperation under the latter, would serve to decrease terror attacks. Therefore, the present work amply demonstrates that a *one-size-fits-all* CT architecture is undesirable, and calls for reviewing the existing CT policy framework in view of the implications of strategic cooperation between terror outfits.

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Appendix 1

The Lagrangian problem is given by:

$$\max_{\{X,A_1,A_2,\lambda,\mu,\gamma_1,\gamma_2\}}L$$

where

$$L = X + \alpha_1 A_1 + \alpha_2 A_2 + \lambda \left[R_1 + R_2 - X - \frac{1}{2} \left(\beta_1 A_1^2 + \beta_2 A_2^2 \right) \right] + \mu X + \gamma_1 A_1 + \gamma_2 A_2$$

The relevant K–T conditions for solving the above problem are:

(i)
$$\frac{\partial L}{\partial X} = 1 - \lambda + \mu = 0$$

(ii)
$$\frac{\partial L}{\partial A_i} = \alpha_i - \lambda \beta_i A_i + \gamma_i = 0; \quad i = 1, 2$$

- (iii) $\lambda \ge 0, \mu \ge 0, \gamma_i \ge 0 (i = 1, 2)$
- (iv) $X \ge 0, A_i \ge 0 (i = 1, 2), R_1 + R_2 \ge X + \frac{1}{2} \left(\beta_1 A_1^2 + \beta_2 A_2^2 \right)$

(v)
$$\mu X = 0, \gamma_i A_i = 0 (i = 1, 2) \text{ and } \lambda \left[R_i + R_j - X - \frac{1}{2} \left(\beta_i A_i^2 + \beta_j A_j^2 \right) \right] = 0$$

In our formulation, $A_i > 0$, and so $\gamma_i = 0 \forall i = 1, 2$ (from (v)). Now consider the following cases:

Case (a) Consider equilibrium with X > 0; this means $\mu = 0$ (see (v)), hence $\lambda = 1$ (from (i)). This leads to cooperative equilibrium (from (ii)):

$$A_{i} = \frac{\alpha_{i}}{\beta_{i}} \equiv A_{i}^{C} \quad \forall i = 1, 2 \text{ with } R_{1} + R_{2} > \frac{1}{2} \frac{\alpha_{1}^{2}}{\beta_{1}} + \frac{1}{2} \frac{\alpha_{2}^{2}}{\beta_{2}}.$$

Case (b) Consider equilibrium with X = 0. This means $\mu \ge 0$, and hence $\lambda = 1 + \mu \ge 1$ (see (v) and (i)). When $\mu = 0$, $\lambda = 1$, and the cooperative equilibrium is given by

$$A_i^C = \frac{\alpha_i}{\beta_i}$$
 $\forall i = 1, 2 \text{ and } R_1 + R_2 = \frac{1}{2} \frac{\alpha_1^2}{\beta_1} + \frac{1}{2} \frac{\alpha_2^2}{\beta_2}$

If $\mu > 0$, then $\lambda > 1$. Hence, $A_i = \frac{\alpha_i}{\lambda \beta_i} < \frac{\alpha_i}{\beta_i}$ $\forall i = 1, 2$ from (ii), and $R_1 + R_2 = \frac{1}{2} (\beta_1 A_1^2 + \beta_2 A_2^2)$ (from (iv)). Then plugging the values of A_1 and into this expression, we get: $\frac{1}{\lambda^2} = \frac{2(R_i + R_j)\beta_i\beta_j}{\alpha_i^2\beta_i + \alpha_i^2\beta_i}$.

Therefore, we get the cooperative solution

$$A_i^C = \sqrt{\frac{2(R_i + R_j)\beta_i\beta_j}{\alpha_i^2\beta_j + \alpha_j^2\beta_i}} \cdot \frac{\alpha_i}{\beta_i} \quad \text{for } i \neq j$$

This solves the cooperative game.

Appendix 2

The first order conditions (FOCs) to the maximization problem given in Eq. (11) are $\frac{dU_i}{dA_i} = \alpha_i + \frac{A_i}{(A_i + A_j)^2}F - \beta_iA_i = 0$, for i = 1, 2 while the second order conditions (SOCs) are.. It is easy to see that the SOCs hold. And finally, the stability and uniqueness condition is $\frac{\partial^2 U_i}{\partial A_i^2} \frac{\partial^2 U_j}{\partial A_j^2} - \frac{\partial^2 U_i}{\partial A_i \partial A_j} \frac{\partial^2 U_i}{\partial A_j \partial A_i} = \left(\beta_i + 2\frac{A_j}{(A_i + A_j)^3}F\right)\left(\beta_j + 2\frac{A_i}{(A_i + A_j)^3}F\right)$ $+ \frac{(A_i - A_j)^2}{(A_i + A_j)^6}F^2 > 0$, which also holds. Therefore, if $R_i > \frac{1}{2}\frac{\alpha_i^2}{\beta_i}$, i = 1, 2; the slope of T_i 's reaction function as obtained from the FOCs is $\frac{dA_i}{dA_j} = -\frac{\frac{\partial^2 U_i}{\partial A_i^2}}{\frac{\partial^2 U_i}{\partial A_i^2}} = \frac{(A_i - A_j)F}{\beta_i(A_i + A_j)^3 + 2A_jF} \gtrless 0$

according as $A_i \ge A_j$, $j \ne i = 1, 2$. This means that the reaction functions are initially sloped positively until they intersect the line of equality $(A_i = A_j)$, and thereafter sloped negatively. On the other hand, if for any outfit T_i , i = 1, 2 we

have $R_i \leq \frac{1}{2} \frac{\alpha_i^2}{\beta_i}$, T_i 's reaction function becomes $A_i = \sqrt{\frac{2R_i}{\beta_i}}$, which is independent of A_i , $j \neq i = 1, 2$.

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